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THEORETICAL ANALYSIS ON J-PARC E31 EXPERIMENT

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Abstract

The research work is to analyze the missing mass and the invariant mass spectra of D (K^-, n) Λ (1405) reaction process which was conducted at J-PARC E31 experiment with 1.0 GeV/c incident momentum of K^-. This reaction is expected to enhance a virtual $\overline{K}N$ scattering process, where a K^- beam kicks a neutron out of the deuteron target in a forward angle and is slowing down to form a Λ(1405) with a residual nucleon. We have calculated the missing mass spectrum D(K^-, n) Y reaction with Green's function method by using YA potential for $\overline{K}N$ interaction. We have also used the $\overline{K}N \rightarrow \pi\Sigma$ coupled channel Yukawa type separable potential to compute the invariant mass spectrum. We observed that the missing mass spectrum of the D (K^-, n) Y reaction at a neutron forward angle has two peaks below the K^-p threshold and above the threshold respectively. The former peak represents Λ (1405) state while the latter is a quasi-free K^-p peak. . We have analyzed the invariant mass spectrum of $D(K^-, n)(\Sigma\pi)^{I-0}$ where final state Σπ is given in $|I=0\rangle$ isospin basis. Our calculated invariant mass spectrum of Σπ shows the prominent peak below the threshold while quasi-free part is largely suppressed. We have also studied the invariant mass spectrum with final Σπ charge states separately which are $\Sigma^+\pi^-$, $\Sigma^-\pi^+$, $\Sigma^0\pi^0$

Key words: virtual $\overline{K}N$ scattering, missing mass spectrum, invariant mass spectrum.

Introduction

An antikaon ($\overline{K}$) and a nucleus may form a bound state (a kaonic nucleus), due to the strong attraction between $\overline{K}$ and nucleon in an isospin I=0 state. Λ(1405) resonance state is nominally accepted as a bound state of

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K^-p system which lies in the continuum region of $\pi\Sigma$, having strangeness S=-1, total charge Q=0, isospin I=0 and spin parity $J^p=1/2^-$.

The updated PDG value of the mass and width of this Λ (1405) resonance state or often known as Λ* is $1405.1^{+1.3}_{-1.0}$ MeV/c^2 and 50.5 ± 2.0 MeV (Particle Data Group K. A. Olive *et al*., (2014)) with 27 MeV binding energy with respect to $\overline{K}N$ threshold.

On the other hand, chiral unitary model claims that Λ (1405) may have two pole structure; one is mainly coupled to $\pi\Sigma$ state and the other is to $\overline{K}N$ state which are located at different positions, (1390-132i) MeV and (1426-32i) MeV, respectively (T. Hyodo and A. Weise, (2008)). As a consequence, the resonance position of the Λ (1405) is 1420 MeV/c^2 and the binding energy is as shallow as 15 MeV.

The mass and width of Λ (1405).resonance were obtained to be 1400.5 ± 4.0 MeV/c^2 and 50.0 ± 2.0 MeV from production of Λ (1405) in K^-p reactions at 4.2 GeV/c (R. J. Hemingway, (1985)) by Dalitz and Deloff (R. H. Dalitz and A. Deloff, (1991)). It is interpreted as a quasi-bound state of $\overline{K}N$ coupled with continuum stat of $\pi\Sigma$.

Esmaili *et al.,*(J. Esmaili, Y. Akaishi and T. Yamazaki, (2010), (2011)) analyzed old bubble-chamber of stopped-K^- on ^{4}He (B. Riley *et al*., (1975)) with a resonance capture process, and found the best-fit value of mass and width for the Λ (1405) are $1405.1^{+1.3}_{-1.0}$ MeV/c^2 and $24.0^{+4.0}_{-3.0}$ MeV.

Maryam *et al*. (M. Hassanvand, Y. Akaishi, T. Yamazaki, (2015)) have calculated the $\Lambda(1405) \rightarrow (\pi\Sigma)^0$ invariant mass spectra produced in the reaction $K^- + p \rightarrow \Sigma^+(1660) + \pi^-$, followed by $\Sigma^+(1660) \rightarrow \Lambda(1405) + \pi^+ \rightarrow \Sigma\pi + \pi^+$, processes at p($K^-$)=4.2 GeV/c.

Many experimentalists and theorists have studied the structure of Λ (1405) with different reactions by using various methods since nearly 1960's. But, the structure of Λ (1405) is still a controversial problem. So, H. Noumi *et al*., proposed an experiment to study Λ (1405) via the D (K^-, n) reaction at J-PARC (E31). In this experiment, missing mass and invariant-mass spectrum of D (K^-, n) at a neutron forward angle were measured. We analyzed both missing mass and the invariant-mass spectrum of D (K^-, n) Λ (1405) reaction process with K^- momentum 1.0 GeV/c is incident upon the deuteron target.

Formulation of differential cross section for D (K^- , n) Λ (1405)

The mathematical expression for spectral function is the most significant factor to determine the reaction cross-section. Therefore, we are going to determine the differential cross section and spectral function for D (K^- , n) Λ (1405) reaction. The differential cross section is defined as the transition rate per incident flux.

$$d^6\sigma = \frac{L^3}{v_0}\frac{2\pi}{\hbar}\sum_n \delta(E_i - E_f^{(n)})(\frac{L}{2\pi})^3 d\vec{k}_n (\frac{L}{2\pi})^3 d\vec{K}\left|T_{fi}^{(n)}\right|^2 \qquad (1)$$

Where, $\frac{L^3}{v_0}$ = incident flux, incident kaon velocity, $v_0 = \frac{\hbar k_0 c^2}{E_0}$,

$\left(\frac{L}{2\pi}\right)^3 d\vec{k}_n \left(\frac{L}{2\pi}\right)^3 d\vec{K}$ is phase space, $\delta(E_i - E_f^n)$ is energy conservation term and , T=transition operator.

We consider the elementary process of reaction is $K^- + D \rightarrow n + \Lambda(1405)$.

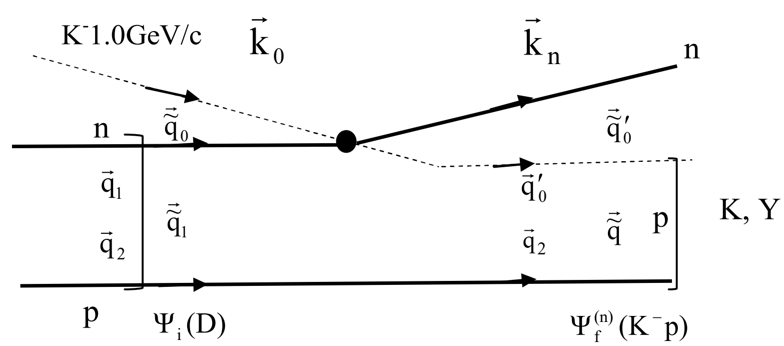

Figure 1. Schematic diagram of the reaction D (K^- , n) Λ (1405) reaction

Transition Matrix Element

We describe, transition matrix for D (K^- , n) Λ (1405) reaction as follows, $T_{fi} = \langle \text{final state} | T | \text{initial state} \rangle$

$$T_{fi} = \left\langle \Psi_f^n \left(K^- p\right), \vec{K}, \vec{k}_n \; \middle| T \middle| \Psi_i(D), \vec{0}, \vec{k}_0 \right\rangle \tag{2}$$

Where, $\Psi_f^n(K^- p)$ involves $\vec{q}_2$ and $\vec{q}_0'$, $\Psi_i(D)$ involes $\vec{q}_1$ and $\vec{q}_2$.

The transition matrix element can be expressed as follows;

$$\left|T_{fi}^{(n)}\right|^2 = \left| \int d\vec{q}_1 \left\langle \Psi_f^{(n)}(K^- p) \middle| \vec{\tilde{q}} \right] \delta(\vec{K} + \vec{k}_n - \vec{k}_0) \left[\vec{\tilde{q}}_0' \middle| t_{K^- n} \middle| \vec{\tilde{q}}_0 \right] \left[\vec{q}_1 \middle| \Psi_i(D) \right\rangle \right|^2 \tag{3}$$

By substituting equation (4) into equation (1), and then we get,

$$\frac{d^2\sigma}{d\cos(\theta)} = \frac{(2\pi)^5}{\hbar^2 k_0 c^2} E_0 k_n^2 dk_n \left| \left\langle t_{K^- n} \right\rangle \right|^2$$

$$\times (-\frac{1}{\pi}) \mathrm{Im} \int d\vec{r}' d\vec{r} f^*(\vec{r}') \left\langle \vec{r}' \middle| \frac{1}{E - H_{K^- p} + i\varepsilon} \middle| \vec{r} \right\rangle f(\vec{r}) \tag{4}$$

Where, $\frac{(2\pi)^5}{\hbar^2 k_0 c^2} E_0 k_n^2 dk_n \left| \left\langle t_{K^- n} \right\rangle \right|^2$ is the kinematical factor and

$(-\frac{1}{\pi}) \mathrm{Im} \int d\vec{r}' d\vec{r} f^*(\vec{r}') \left\langle \vec{r}' \middle| \frac{1}{E - H_{K^- p} + i\varepsilon} \middle| \vec{r} \right\rangle f(\vec{r})$ is the spectral function S(E).

In this spectral function equation, $\left\langle \vec{r}' \middle| \frac{1}{E - H_{K^- p} + i\varepsilon} \middle| \vec{r} \right\rangle$ is the Green's function.

Green's function can be expressed by coordinate representation

$$G(\vec{r}', \vec{r}) = \left\langle \vec{r}' \middle| \frac{1}{E - H_{K^- p} + i\varepsilon} \middle| \vec{r} \right\rangle \tag{5}$$

(H_{K^-p} is Hamiltonian of K^- and p nucleus or K^-p system)

It is satisfies the following equation,

$$\left(E - H_{K^-p}\right)G^+\left(\vec{r}', \vec{r}\right) = \left\langle \vec{r}' \middle| 1 \middle| \vec{r} \right\rangle = \delta\left(\vec{r}' - \vec{r}\right) \tag{6}$$

Radial part of Green's function $G^+(r', r)$ satisfies the following equation,

$$[k^2 + \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \widetilde{U}(r)]G_\ell^{\ +}(r', r) = \frac{2\mu}{\hbar^2}\delta(r' - r) \tag{7}$$

$$k = \sqrt{\frac{2\mu E}{\hbar^2}}, \widetilde{U}(r) = \frac{2\mu}{\hbar^2} V_{K^-p}(r)$$

Green's function $G_\ell^{\ +}(r', r)$ is divided into two regions $G_{\ell 1}^{\ +}(r', r)$ and $G_{\ell 2}^{\ +}(r', r)$ with,

$G_{\ell_1}^{+}(r', r) = C_1 u_\ell^{(0)}(r)$ where, $(0 \langle r \langle r')$

$G_{\ell_2}^{(+)}(r', r) = C_1 u_\ell^{(+)}(r)$ where, $(r' \langle r \langle \infty)$

$u_\ell^{(0)}(r)$ and $u_\ell^{(+)}(r)$ satisfies,

$[k^2 + \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \widetilde{U}(r)]u_\ell^{(0)}(r) = 0$ with boundary condition $u_\ell^{(0)}(0) = 0$.

$[k^2 + \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \widetilde{U}(r)]u_\ell^{(+)}(r) = 0$ with boundary condition $u_\ell^{(+)}(r \to \infty) = krh_\ell^{+}(kr)$.

According to the continuity of Green's function,

$$C_1 u_\ell^{(0)}(r)\big|_{r'} = C_2 u_\ell^{(+)}(r)\big|_{r'} \tag{8}$$

Discontinuity of $\frac{dG}{dr}$ gives

$$C_1 u_\ell^{(0)'}(r)\Big|_{r'} - C_2 u_\ell^{(+)'}(r)\Big|_{r'} = \frac{2\mu}{\hbar^2} \tag{9}$$

We have

$$G_\ell^{(+)}(\vec{r}',\vec{r}) = \frac{2\mu}{\hbar^2}\sum_{\ell=0}^{\infty}\sum_M Y_{\ell M}(\hat{r}')\frac{u_\ell^{(+)}(r_>)u_\ell^{(0)}(r_<)}{W\left(u_\ell^{(0)},u_\ell^{(+)}\right)}Y^*_{\ell M}(\hat{r}) \tag{10}$$

The missing mass spectrum from this reaction is calculated by using Green's function method. It can express as follows;

$$\frac{d^2\sigma}{dY d\cos(\theta)} = \frac{(2\pi)^5}{\hbar^4 k_0}E_0 k_n^2\left|\langle t_{K^-n}\rangle\right|^2 \frac{Y}{(1+\frac{E_Y}{E_n})k_n - k_0\cos(\theta)}$$

$$\times\frac{2\mu}{\hbar^2}\left(\frac{-1}{\pi}\right)\mathrm{Im}\left[\sum_\ell(2\ell+1)\int dr dr' j^*_{\ell'}(Qr')u_i^*(r')\frac{u_\ell^{(+)}(r_>)u_\ell^{(0)}(r_<)}{W\left(u_\ell^{(0)},u_\ell^{(+)}\right)}j_\ell(Qr)u_i(r)\right] \tag{11}$$

Calculation of reaction cross-section with separable potential for Σπ invariant- mass spectrum from D (K⁻, n) reaction

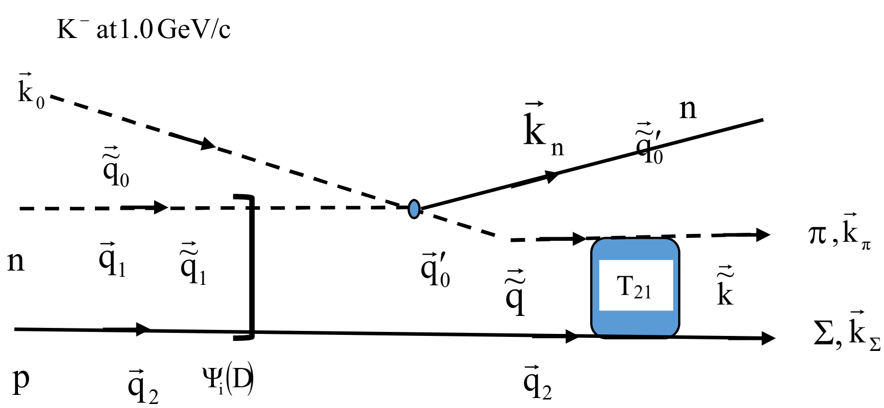

Figure 2. Schematic diagram of D (K⁻, n) $(\Sigma\pi)^0$ decay process

We can express transition matrix element and differential cross-section for Σπ decay process as follows;

$$T_{fi} = \left[\vec{k}_\pi, \vec{k}_\Sigma, \vec{k}_n \middle| T \middle| \vec{k}_0, \vec{0}, \psi_i \right\rangle \tag{12}$$

$$d^9\sigma = \frac{L^3}{v_0}\frac{2\pi}{\hbar}\sum_n \delta(E_i - E_f^{(n)})(\frac{L}{2\pi})^3 d\vec{k}_n (\frac{L}{2\pi})^3 d\vec{K}(\frac{L}{2\pi})^3 d\vec{\tilde{k}} \left|T_{fi}^{(n)}\right|^2 \tag{13}$$

$$\frac{d^2\sigma}{dYc^2 d\cos\theta_n} = 2\left(\frac{2\pi}{\hbar c}\right)^6 \left|\langle t_{K^-n}\rangle\right|^2 \frac{E_0}{k_0}\frac{k_n^2 E_Y}{\left(1+\frac{E_Y}{E_n}\right)k_n - k_0\cos\theta_n}$$

$$\times\left|F_d(Q)\right|^2 \left|g(\tilde{k})\,T_{21}(Y)\right|^2 \frac{\tilde{E}_\pi \tilde{E}_\Sigma}{\tilde{E}_\pi + \tilde{E}_\Sigma}\tilde{k}(Y) \tag{14}$$

In equation (2.3), T_{21} is the transition matrix element for $\bar{K}N$ -$\Sigma\pi$ coupled-channel system. So, we have used the $\bar{K}N \rightarrow \pi\Sigma$ coupled channel Yukawa type separable potential to compute the invariant mass spectrum.

$\bar{K}N$ -Σπ coupled-channel system

We treat the K^-p quasi-bound state as a Feshbach resonance (H. Feshbach, Ann. Phys. (1958), (1962)) embedded in the Σπ continuum by using Akaishi-Myint-Yamazaki's (AMY) phenomenological model. In the AMY model, we use a set of separable potentials with a Yukawa-type form factor for the coupled system of $\bar{K}N$ and Σπ channels.

$$\left\langle \vec{k}_i' \middle| v_{ij} \middle| \vec{k}_j \right\rangle = g\left(\vec{k}_i'\right)U_{ij}\, g\left(\vec{k}_j\right), \left\langle \vec{\tilde{k}} \middle| t_{21} \middle| \vec{\tilde{q}} \right\rangle = g(\vec{\tilde{k}})\,T_{21}(Y)\,g(\vec{\tilde{q}})$$

$$g(\vec{\tilde{k}}) = \frac{\Lambda^2}{\Lambda^2 + \vec{\tilde{k}}^2}, U_{ij} = \frac{1}{\pi^2}\frac{\hbar^2}{2\sqrt{\mu_i \mu_j}}\frac{1}{\Lambda}s_{ij}$$

Where, i (j) stands for the $\bar{K}N$ channel, 1, or the Σπ channel, 2, μ_i (μ_j) is the reduced mass of channel i (j), and s_{ij} are non-dimensional strength parameters. Then, a complex potential with the following strength is derived analytically;

$$s_1^{opt}(E) = s_{11} - s_{12}\frac{\Lambda^2}{(\Lambda - i\kappa_2)^2 + s_{22}\Lambda^2}s_{21}, E + \Delta Mc^2 = \frac{\hbar^2}{2\mu_2}\kappa_2^2$$

$\Delta M = m_{K^-} + M_p - m_{\pi^-} - M_{\Sigma^+} = 103\,\mathrm{MeV/c^2}$ is the threshold mass difference,

κ_2 is a complex momentum in the $\Sigma\pi$ channel.

In this model, we use s_{22}=-0.66, which gives U_{22}/U_{11}=4/3 for Λ (1405) as in a "chiral model", and Λ=3.90/fm. In this mode, the loop integral is

$$\tilde{G}(E) = \int d\vec{q}\,' d\vec{q}\,\frac{\Lambda^2}{\Lambda^2 + \vec{q}\,'^2}\left\langle \vec{q}\,' \left| \frac{1}{E - H + i\varepsilon} \right| \vec{q} \right\rangle \frac{\Lambda^2}{\Lambda^2 + \vec{q}^{\,2}}$$

The transition –matrix of the two coupled channels, $\bar{K}N$ (1) and $\Sigma\pi$ (2), obeys the following equation;

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{21} \end{pmatrix} + \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{21} \end{pmatrix} \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{21} \end{pmatrix}$$

The solutions of each matrix are;

$$T_{ii} = \frac{1}{1 - U_{ii}^{opt}G_i}U_{ii}^{opt},\quad T_{ji} = \frac{1}{1 - U_{jj}^{opt}G_j}U_{ji}^{opt}$$

The generalized optical potentials are;

$$U_{ii}^{opt} = U_{ii} + U_{ij}\frac{G_j}{1 - U_{jj}G_j}U_{ji},\quad U_{ji}^{opt} = U_{ji}\frac{1}{1 - U_{ii}G_i}$$

It should be noticed that the two-channel coupled equation is divided into four single-channel effective equations without any approximation by the use of the optical potentials. The invariant mass spectrum from $K^- + D \rightarrow n + \Lambda(1405) \rightarrow n + (\Sigma\pi)^{(0)}$ is calculated by using separable potential.

Results and Discussions

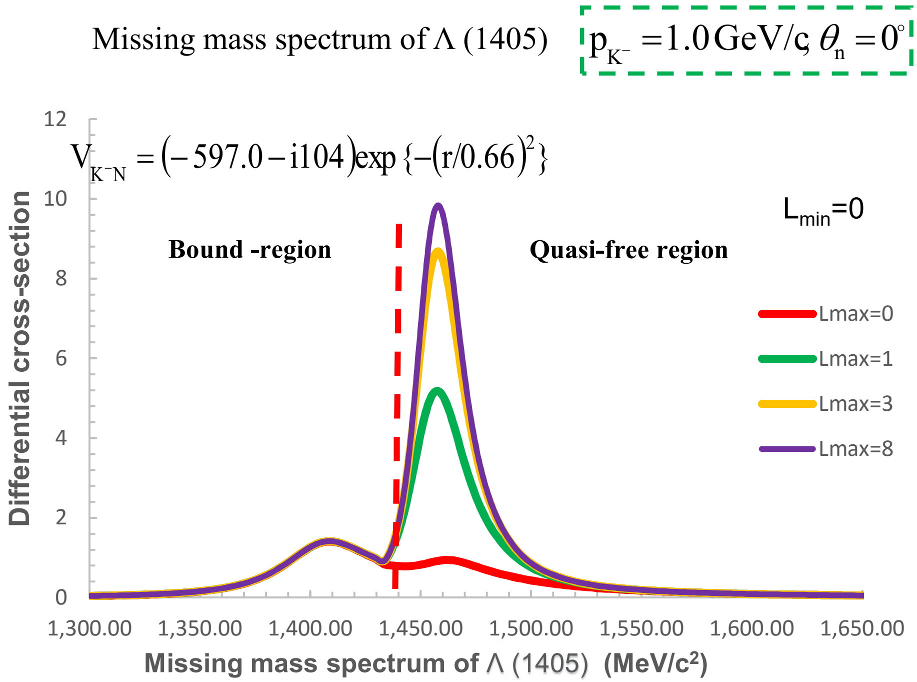

Figure 3. Missing mass spectrum of Λ (1405) with different angular momentum. Red color dotted line represents the $\bar{K}N$ threshold.

The calculated missing mass spectrum of the $D(K^-, n)$ reaction for various angular momentum distribution as shown in fig.(3). Here, we used the YA potential for K-P bound system with V_0, W_0) = (-597.0 MeV-104.0 MeV). From the calculated missing mass spectrum, it can be seen that $\bar{K}N$ bound state is mainly contributed by L=0 while in continuum region, higher angular momentum contributions dominate. According to our analysis, the mass and width of Λ (1405) is about 1409.0 MeV/ C^2 and 47 MeV, respectively.

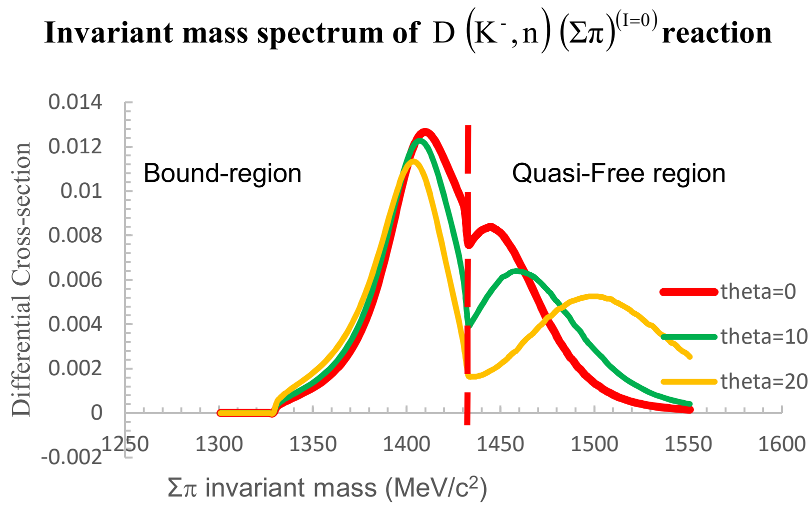

Figure 4. $\Sigma\pi$ invariant mass spectrum with θ_n=0°, 10°, 20°. Red color dotted line represents the $\bar{K}N$ threshold.

Figure (4) shows the $\Sigma\pi$ invariant mass spectrum with emitted neutron angles, θ_n=0°, 10°, 20°. We observe that the peak position in bound state region remain unchanged while the quasi-free peaks shift towards the higher mass region with an increase angle, θ_n . From the calculated invariant mass spectrum, the mass and width of Λ (1405) is about 1406.0 MeV/C^2 and 52 MeV, respectively. The value of mass and width of Λ (1405) which obtained from missing mass spectrum is nearly consistent that of $\Sigma\pi$ invariant mass spectrum.

Deuteron size effect upon the Σπ invariant mass spectrum

We studied the deuteron size effect upon the $\Sigma\pi$ invariant mass spectrum by changing the size parameter 'a' of deuteron wave function, which is given by $\psi_i = \left(\frac{a}{2\pi}\right)^{\frac{3}{4}} e^{-\frac{1}{4}a\vec{r}^2}$, where smaller 'a' gives larger size. We have arbitrarily varied the deuteron size by multiplying the size parameter 'a' with a multiplicative factor 'f '. The results are shown in figure (5).

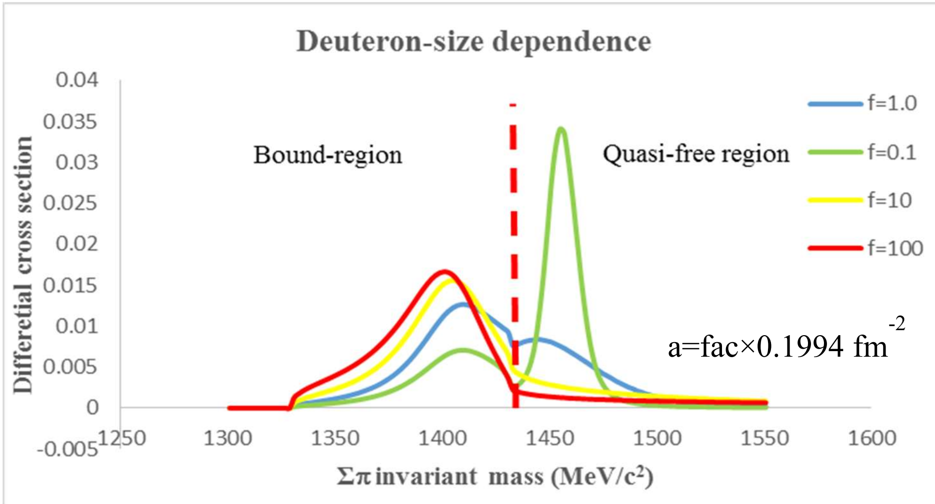

Figure.5 Σπ invariant mass spectrum depends on deuteron size. Red color dotted line represents the $\overline{K}N$ threshold.

We investigated the deuteron size effect for the cases; f = 0.1, 1, 10 and 100 respectively which are shown in the above figure. From this investigation, it is found that the smaller the deuteron size (larger 'a'), the larger the differential cross-section.

Invariant mass-spectrum of D (K^-, n) Σπ charge basis

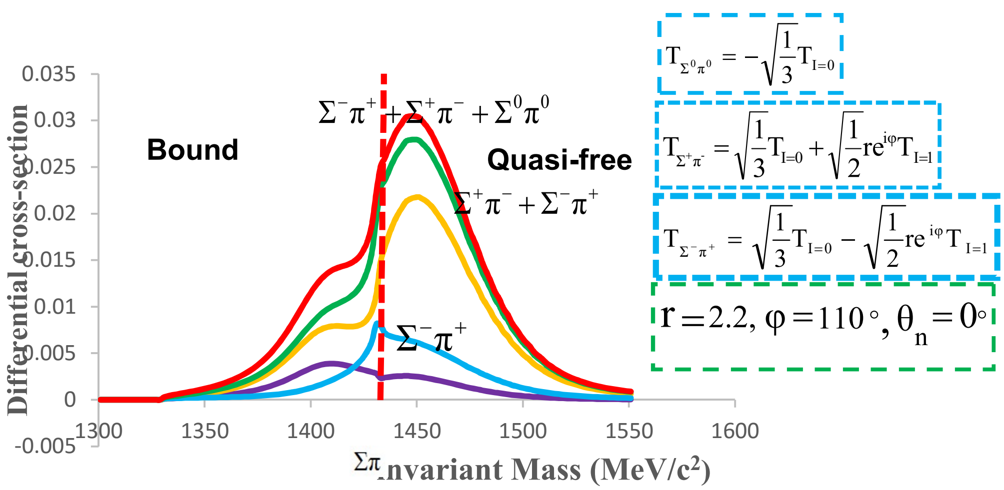

Figure 6. Invariant mass spectrum with final Σπ charge states. . Red color dotted line represents the $\overline{K}N$ threshold

E 31 experimental data compare with our results and Chiral unity model results

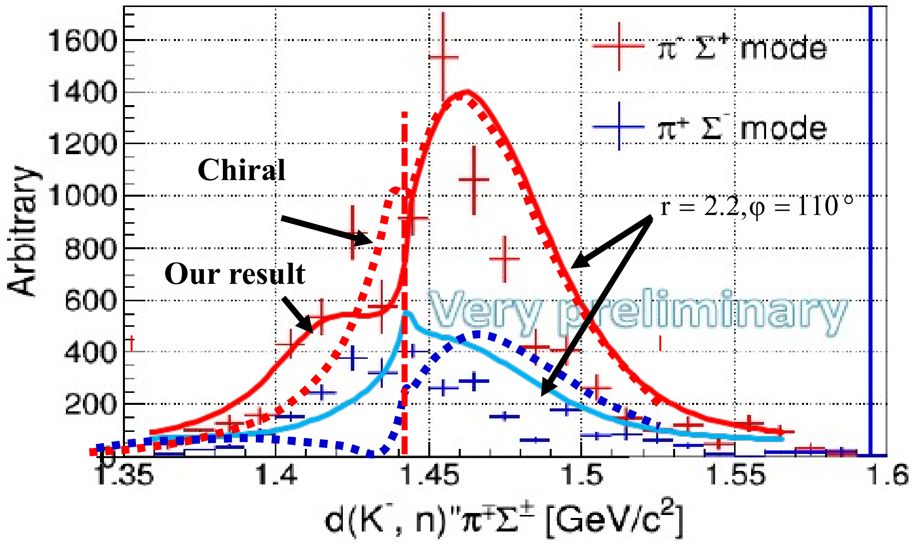

Figure 7. E 31 experimental data compare with our results. . Red color dotted line represents the $\overline{K}$ N threshold

Conclusion

We have analyzed the missing mass and the invariant-mass spectrum of D (K^- , n) Λ (1405) reaction at the incident momentum of K^- 1.0GeV/c. The calculated results of invariant mass spectrum of $\Sigma^+\pi^-$and $\Sigma^-\pi^+$are in good agreement with the values obtained from a very preliminary experimental data of $D(K^-,n)(\Sigma\pi)^{(0)}$ charge states. Moreover, the calculated results are also consistent with the updated PDG (2016) value

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